Parabolic Antennas

What is a parabolic antenna?

A parabolic antenna is used for microwave radio communications. It is often referred to as a dish antenna. It consists of a parabolic reflector which collects and concentrates an incoming parallel beam of radio waves and focuses them onto the actual antenna placed at its focal point or focus. The actual antenna at the focus is sometimes referred to as the antenna feed. See Figure 1. In this paper, the term antenna will refer to the actual antenna at the focal point of the the parabolic reflector and parabolic antenna refers to its antenna together with its parabolic reflector.

Satellite television dish antennas are an example of popular parabolic microwave antennas. A parabolic antenna is similar to a reflecting telescope where a parabolic concave mirror gathers incoming light and focuses it into the eyepiece. Another optical example is the automobile headlight. A parallel beam of light is reflected off the parabolic mirror behind the light bulb placed at its focus.

The following terminology is used in describing a parabolic reflector. The focus is where all the incoming radio waves are concentrated. The vertex is the innermost point at the center of the parabolic reflector. The focal length of a parabola is the distance from its focus to its vertex. The aperture of a parabolic reflector is its opening and is described by its diameter. Also of interest is the depth of the parabolic reflector which is discussed below. See Figure 2.

The two dimensions that describe a parabolic reflector are the focal length $f$ and the diameter $D$ of its aperture. The industry practice is to use the $f/D$ ratio to specify the shape of the parabolic reflector and the diameter $D$ of its aperture. For a given parabolic reflector, the focal length $f$ is easily obtained by multiplying its $f/D$ ratio by its diameter $D$. 
Design formulas for a parabolic reflector

The following formulas are useful for designing a parabolic reflector. The derivation of the formulas are given in Appendix: Derivation of Equations and Formulas.

Equation of a parabola

The general equation of a parabola in terms of its focal length $f$ is

$$ y = ax^2 \quad (1) $$

where

$$ a = \frac{1}{4f} \quad (2) $$

Equation (1) useful for laying out a parabola on large sheet of graph paper.

Depth of a parabolic reflector

In designing a parabolic reflector, it is often convenient to use its depth $d$ instead of its focal length. The formula for obtaining the depth is

$$ d = \frac{D^2}{16f} \quad (3) $$

Conversely, given a parabolic dish and its measurements for the diameter $D$ and the depth $d$, then its focal length $f$ is obtained with

$$ f = \frac{D^2}{16d} \quad (4) $$
Calculating \( f/D \) given the beamwidth of an antenna

An antenna placed at the focal point of a parabolic reflector is said to *illuminate* the parabolic reflector. The antenna has a *beamwidth* which is the how wide an angle the antenna would make if it were radiating a beam of radio waves. The beamwidth is a property of the antenna itself and is the same regardless if the antenna is used for receiving or transmitting. In designing a parabolic antenna, the antenna needs to properly illuminate its parabolic reflector; that is, the beamwidth of the antenna needs to match the \( f/D \) ratio of the parabolic reflector. Otherwise, the antenna of an *over illuminated* parabolic reflector would receive a noise from behind the parabolic reflector. Likewise, an *under illuminated* parabolic reflector does not use its total surface area to focus a signal on its antenna.

The matching \( f/D \) ratio of a parabolic reflector is easily calculated using

\[
\frac{f}{D} = \frac{1}{4 \tan \left( \frac{\theta}{4} \right)}
\]

where \( \theta \) is the beamwidth in degrees.

**Length of a parabolic segment**

The formula for calculating the length \( L \) of a parabolic segment from the vertex to the edge is

\[
L = \frac{\ln \left( \sqrt{a^2 D^2 + 1} + aD \right)}{4a} + \frac{D \sqrt{a^2 D^2 + 1}}{4}
\]

where \( a \) is defined in equation (2).

This formula is useful for calculating the length of a support rib on the outside of the parabolic reflector.
**Surface area of a parabolic reflector**

The surface area of a parabolic reflector is calculated with this formula

\[ S = \pi \frac{(a^2 D^2 + 1)^{3/2} - 1}{6a^2} \]  

(7)

where \( a \) is defined in equation (2). This formula is useful for estimating the amount of reflective material you need for your parabolic reflector.

**Gain of a parabolic reflector**

Using the formula for the area of a circle, the area of the aperture of a parabolic reflector is

\[ A = \frac{\pi D^2}{4} \]  

(8)

This area is used in calculating the gain of a parabolic reflector. The gain \( G \) of a parabolic reflector is proportional to the ratio of the area of the aperture to the square of the wavelength \( \lambda \) of the incoming radio waves

\[ G = 10 \log_{10} \left( \eta \frac{4 \pi A}{\lambda^2} \right) \]  

(9)

\( \eta \) is the efficiency of the parabolic reflector and has a practical value of 50%. In electrical engineering, it is common practice to express gain ratios such as \( G \) in terms of decibels which is 10 times the common logarithm of the gain ratio. The units of \( G \) is in dbi\(^1\).

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\(^1\) The gain of a parabolic antenna is compared to a theoretical isotropic antenna which has a gain of zero dbi.
A design example

A Montgomery College electrical engineering student decides to build a small radio telescope in her parent’s backyard. She wants to design a five meter parabolic antenna 1420 MHz for the project after buying an antenna feed with a beamwidth of 105 degrees. Her calculations are shown in Appendix: A Design Example Spreadsheet. Note that the length of a support rib is about four percent larger than the radius of the aperture and the surface area of the parabolic reflector is about six percent larger than the area of its aperture. Also, note that it is not worth doubling the diameter of the parabolic antenna since it only gives just another six decibels of gain.
References


